Effect of dissipation on quantum coherence

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The effect of dissipation on a macroscopic superposition of quantum states is studied with use of a Markovian master-equation approach. It is shown that a superposition of two states is reduced to a mixture at a rate proportional to the separation between the two states. This underlines the difficulty of observing a superposition of macroscopic quantum states in practice.

I. INTRODUCTION

There is recently considerable interest in the phenomena of quantum tunneling and quantum coherence.[1] Since no system may be completely isolated from its environment, particular attention must be paid to the influence of dissipation on these phenomena. The quantum effects of interest involve a linear superposition of macroscopically different quantum states. The question that arises is how does such a macroscopic superposition survive the interaction with a heat bath? There are several examples of experimental interest. These include SQUID (superconducting quantum interference device) rings, and current-biased Josephson junctions where experiments in the regime of quantum behavior are being performed[2] and in optical bistability where experiments investigating quantum features are planned.

An interesting discussion on the influence of damping on quantum interference has recently been given by Caldeira and Leggett.[3] They have included the effects of dissipation by an influence functional technique which includes both the cases of strong and weak damping. In this paper we wish to show how similar conclusions may be reached using master-equation techniques. The master-equation approach, though valid in general only for weak damping (as is the case for quantum optical systems), allows for some exact solutions which show clearly the effects of dissipation on macroscopic superpositions of quantum states. A related discussion on the decay of quantum coherence in an N-particle system interacting with a Markovian heat bath has recently been given by Alicki.[4]

II. QUANTUM INTERFERENCE IN A HARMONIC OSCILLATOR POTENTIAL

As an illustrative example we consider the interference produced by a superposition of two coherent states in a harmonic oscillator. We prepare initially the harmonic oscillator in the state

$$|\psi\rangle = |\alpha_1\rangle + |\alpha_2\rangle \; . \tag{2.1}$$

The density operator of the system at time t is

$$\rho(t) = \sum_{i,j=1}^{2} |\alpha_i(t)\rangle\langle\alpha_j(t)| \; , \tag{2.2}$$

where $\alpha_i(t) = |\alpha_i e^{-i\omega t}\rangle$. The probability density at position x may be evaluated using

$$\langle x|\alpha\rangle = \left[\frac{\omega}{\pi\hbar}\right]^{1/4} \exp\left[-\frac{\omega}{2\hbar}x^2 + \left[\frac{2\omega}{\hbar}\right]^{1/2}\alpha x - \tfrac{1}{2}|\alpha|^2 - \tfrac{1}{2}\alpha^2\right] \tag{2.3}$$

we can write

$$\langle x|\rho(t)|x\rangle = I_+^2 + I_-^2 + 2I_+I_-\cos\theta(t) \; , \tag{2.4}$$

where for the case where the initial excitations are of equal amplitude and opposite phase ($\alpha_1 = -\alpha_2 = \alpha$ and α is assumed to be real)

$$I_\pm = \left[\frac{\omega}{\pi\hbar}\right]^{1/4} \exp\left\{-\left[\left[\frac{\omega}{\hbar}\right]^{1/2} x \pm \sqrt{2}\alpha\cos(\omega t)\right]^2\right\} \tag{2.5}$$

and

$$\theta(t) = 2\left[\frac{2\omega}{\hbar}\right]^{1/2} \sin(\omega t)\alpha x \; .$$

This is a simple example of quantum interference where the two wave packets initially separated will produce interference fringes as they cross over. We shall now investigate what happens to the interference terms in the presence of damping.

A. Amplitude damping

We shall first consider a coupling to the reservoir where the amplitude of the harmonic oscillator is damped. This may be described by the Hamiltonian

$$H = \hbar\omega a^\dagger a + a\Gamma_R^\dagger + a^\dagger\Gamma_R \; , \tag{2.6}$$

where λ is the damping constant. We have taken the temrotating-wave approximation has been made in the coupling to the reservoir. In the Born and Markov approximations the reduced density operator for the harmonic oscillator in the interaction picture obeys the equation[5]

$$\frac{\partial\rho}{\partial t}=\frac{\lambda}{2}(2a\rho a^{\dagger}-a^{\dagger}a\rho-\rho a^{\dagger}a)\ , \tag{2.7}$$

where λ is the damping constant. We have taken the temperature of the reservoir to be zero ($T=0$). Finite temperature effects and interactions without the rotating-wave approximation are considered in Sec. III.

The solution for $\rho(t)$ is given by[6]

$$\rho(t)=\sum_{m=0}^{\infty}N_t(m)\rho(0)\ , \tag{2.8}$$

where

$$N_t(m)=\int_0^t dt_m\int_0^{t_m}dt_{m-1}\cdots\int_0^{t_2}S_{t-t_m}JS_{t_m-t_{m-1}}\times\cdots JS_{t_1}\rho(0)\ , \tag{2.9}$$

where

$$J\rho=\lambda a\rho a^{\dagger}\ , \tag{2.10}$$

$$S_t\rho=\exp\left[-\frac{\lambda t}{2}a^{\dagger}a\right]\rho\exp\left[-\frac{\lambda t}{2}a^{\dagger}a\right]. \tag{2.11}$$

Now using the property

$$e^{-\lambda a^{\dagger}at/2}\,|\,\alpha\rangle=\exp\left[-\frac{|\,\alpha\,|^2}{2}(1-e^{-\lambda t})\right]|\,\alpha e^{-\lambda t/2}\rangle \tag{2.12}$$

one can show

$$S_t\,|\,\alpha\rangle\langle\beta\,|=\exp\left[-\frac{|\,\alpha\,|^2}{2}(1-e^{-\lambda t})-\frac{|\,\beta\,|^2}{2}(1-e^{-\lambda t})\right]\times|\,\alpha e^{-\lambda t/2}\rangle\langle\beta e^{-\lambda t/2}\,|\ . \tag{2.13}$$

Thus the time development of an arbitrary initial element $|\,\alpha\rangle\langle\beta\,|$ is

$$(\,|\,\alpha\rangle\langle\beta\,|\,)_t=\exp\left[-\frac{|\,\alpha\,|^2}{2}(1-e^{-\lambda t})-\frac{|\,\beta\,|^2}{2}(1-e^{-\lambda t})\right]|\,\alpha e^{-\lambda t/2}\rangle\langle\beta e^{-\lambda t/2}\,|\times\left[\sum_{m=0}^{\infty}(\lambda\alpha\beta^*)^m\int_0^t e^{-\lambda t_m}dt_m\int_0^{t_m}dt_{m-1}e^{-\lambda t_{m-1}}\cdots\int_0^{t_2}dt_1e^{-\lambda t_1}\right]. \tag{2.14}$$

Using time ordering this becomes

$$\begin{aligned}(\,|\,\alpha\rangle\langle\beta\,|\,)_t&=\exp\left[-\frac{|\,\alpha\,|^2}{2}(1-e^{-\lambda t})-\frac{|\,\beta\,|^2}{2}(1-e^{-\lambda t})\right]|\,\alpha e^{-\lambda t/2}\rangle\langle\beta e^{-\lambda t/2}\,|\sum_{m=0}^{\infty}\frac{(\lambda\alpha\beta^*)^m}{m!}\left[\int_0^t e^{-\lambda t'}dt'\right]^m\\&=\exp\left[(e^{-\lambda t}-1)\left[\frac{|\,\alpha\,|^2}{2}+\frac{|\,\beta\,|^2}{2}-\alpha\beta^*\right]\right]|\,\alpha e^{-\lambda t/2}\rangle\langle\beta e^{-\lambda t/2}\,|\\&=\langle\alpha\,|\,\beta\rangle^{1-e^{-\lambda t}}|\,\alpha e^{-\lambda t/2}\rangle\langle\beta e^{-\lambda t/2}\,|\ .\end{aligned} \tag{2.15}$$

Thus for an initial density operator

$$\rho(0)=N(\,|\,\alpha\rangle\langle\alpha\,|+|\,\beta\rangle\langle\beta\,|+|\,\alpha\rangle\langle\beta\,|+|\,\beta\rangle\langle\alpha\,|\,)\ , \tag{2.16}$$

since $N_t(m)$ is linear, the density operator at time t is

$$\rho(t)=N\sum_{\gamma,\gamma'=\alpha}^{\beta}\langle\gamma\,|\,\gamma'\rangle^{1-e^{-\lambda t}}|\,\gamma e^{-\lambda t/2}\rangle\langle\gamma' e^{-\lambda t/2}\,|\ . \tag{2.17}$$

One sees that the off-diagonal elements of the density matrix in the coherent-state basis are dephased by the factor $\langle\alpha\,|\,\beta\rangle^{1-e^{-\lambda t}}$. The greater the distance between the two itial states, the more rapidly the off-diagonal elements are dephased. A physical explanation for the rapid dephasing may be given by a consideration of the state of the coupled system plus environment before and after one quantum is lost from the system to the environment.[3]

We note that in general for an arbitrary density operator which may be expressed in an off-diagonal coherent-state representation[7]

$$\rho(0)=\int P(\alpha,\alpha')\frac{|\,\alpha\rangle\langle\alpha'\,|}{\langle\alpha'\,|\,\alpha\rangle}d\mu(\alpha,\alpha') \tag{2.18}$$

the solution for the density operator at time t is

$$\rho(t)=\int P(\alpha,\alpha')\frac{|\,\alpha e^{-\lambda t/2}\rangle\langle\alpha' e^{-\lambda t/2}\,|}{\langle\alpha' e^{-\lambda t/2}\,|\,\alpha e^{-\lambda t/2}\rangle}d\mu(\alpha,\alpha')\ . \tag{2.19}$$

Examples of quantum states which are not expressible in terms of a tempered distribution using the diagonal P representation and require the off-diagonal representation

(2.18) are the squeezed states and the number states.

If we now consider the interference experiment discussed earlier in the presence of damping we find

$$\langle x \mid \rho(t) \mid x \rangle = I_+ + I_- + 2e^{-|\alpha|^2(1-e^{-\lambda t})} I_+ I_- \cos\theta(t) , \tag{2.20}$$

where I_+, I_- and $\rho(t)$ are given by Eq. (2.5) with $\alpha \rightarrow \alpha e^{-\lambda t/2}$. The amplitude of oscillation is damped by the interaction with the environment at a rate $\lambda/2$. The interference term, however, is damped at a rate $e^{-|\alpha|^2(1-e^{-\lambda t})}$. That is, the greater the initial separation of the two wave packets the more rapidly the coherence between them is damped. Hence a quantum state prepared in a macroscopic superposition is very rapidly reduced to a mixed state.

This has important implications in the quantum theory of measurement. Zurek[8,9] has shown that the action of the environment on a quantum system can often be regarded as a repeated measurement of the pointer observable Λ of that system. The pointer observable Λ commutes with the system environment interaction Hamiltonian H_{se}

$$[\Lambda, H_{se}] = 0 . \tag{2.21}$$

The interaction with the environment singles out a preferred pointer basis in the Hilbert space of the quantum system. The pointer basis consists of the eigenspaces of the pointer observable Λ. When Λ commutes also with the self-Hamiltonian of the system $[H_{se}, \Lambda] = 0$ it is a quantum nondemolition observable[10] of the measurement performed by the environment.

In the example considered above, we may consider the measuring apparatus to play the role of the environment which measures the complex amplitude of the oscillator via the coupling Eq. (2.6). The initial state of the oscillator has been prepared in a superposition of two eigenstates of a by the first stage of the measurement. The interaction with the environment rapidly reduces this coherent superposition to a mixture. That is, the reduction of the wave packet is accomplished on a very short time scale through the interaction with the measuring apparatus. The concept of a pointer basis has an approximate validity here owing to the coupling of the non-hermitian operator a to the environment. In Sec. II B we shall consider an example which has an exact pointer basis.

B. Phase damping

As another example we consider an interaction with the environment via the number operator of the oscillator. Thus, we consider the Hamiltonian

$$H = \hbar\omega a^\dagger a + a^\dagger a \Gamma . \tag{2.22}$$

With this coupling there is no energy damping; there is, however, a phase damping. In this case, the environment may be considered as making a measurement of the number of quanta in the system. The number operator $a^\dagger a$ is an exact pointer observable of this Hamiltonian since $[a^\dagger a, H] = 0$. It is also a quantum nondemolition observable. The number states $|n\rangle$ are the pointer basis of the system.

The master equation for the reduced density operator of the system in the interaction picture for a finite temperature bath is

$$\frac{\partial \rho}{\partial t} = \frac{\lambda}{2}(2a^\dagger a \rho a^\dagger a - \rho a^\dagger a a^\dagger a - a^\dagger a a^\dagger a \rho) , \tag{2.23}$$

where $\lambda = \lambda' kT$, where λ' is the damping constant.

Substituting

$$\rho(t) = \sum \rho_{mn}(t) \mid n \rangle \langle m \mid$$

into Eq. (2.23) we obtain

$$\frac{\partial}{\partial t}\rho_{mn}(t) = -\frac{\lambda}{2}(n-m)^2 \rho_{mn}(t) \tag{2.24}$$

which has the solution

$$\rho_{mn}(t) = e^{-\lambda(n-m)^2 t/2}\rho_{mn}(0) . \tag{2.25}$$

Thus the coherence between a superposition of two different number states is damped by the factor $\exp[-\lambda(n-m)^2 t/2]$. We shall consider a system where such a decay of coherence between a superposition of number states may possible by observed. A coherent state may be expressed as a superposition of number states as follows:

$$\mid \alpha \rangle = e^{-|\alpha|^2/2} \sum \frac{\alpha^n}{(n!)^{1/2}} \mid n \rangle . \tag{2.26}$$

Thus the density operator for an initial coherent state subject to damping given by Eq. (2.23) would evolve into

$$\rho(t) = e^{-|\alpha|^2} \sum \frac{\alpha^n (\alpha^*)^m}{(n!)^{1/2}(m!)^{1/2}} e^{-\lambda(m-n)^2 t/2} \mid n \rangle \langle m \mid . \tag{2.27}$$

The decay of coherence between different number states may be probed by measuring the kth moment of amplitude

$$\begin{aligned} \langle a^k(t) \rangle &= \mathrm{Tr}[\rho(t) a^k] \\ &= e^{-|\alpha|^2} \frac{\alpha^n (\alpha^*)^{n+k}}{(n!)^{1/2}[(n+k)!]^{1/2}} e^{-\lambda k^2 t/2} . \end{aligned} \tag{2.28}$$

An optical experiment which would simulate the above conditions would involve passing a laser beam through a medium with a fluctuating refractive index, followed by phase-sensitive measurements of the kth moment of the field amplitude.

C. Other reservoir couplings

It is clear from Sec. II B that if one has a pointer observable which is also a constant of the motion, the system will rapidly approach a state which is diagonal in the pointer basis. For example, one may couple the quadrature phase amplitude X_1 ($a = X_1 + iX_2$) of a harmonic oscillator to the environment. This is a quantum-nondemolition-type coupling advocated for detectors of gravitational radiation.[10] In this case the interaction Hamiltonian is

$$H = X_1 \Gamma \ . \tag{2.29}$$

The metric elements of ρ in the pointer basis $|X_1\rangle$ decay as

$$\langle X_1' | \rho(t) | X_1 \rangle = N e^{-\lambda(X_1 - X_1')^2 t/2} \langle X_1' | \rho(0) | X_1 \rangle \ . \tag{2.30}$$

Alternatively, one may consider a system of N spin-$\frac{1}{2}$ particle or N two-level atoms, with total angular momentum $\mathbf{J} = \sum_{i=1}^{N} \sigma_i$. If the coupling to the reservoir is via J_z

$$H = \hbar\omega J_z + J_z \Gamma \ . \tag{2.31}$$

Then the off-diagonal matrix elements in the pointer basis $|m\rangle$ (eigenstates of J_z) decay as

$$\langle m | \rho(t) | n \rangle = N e^{-\lambda(m-n)^2 t/2} \langle m | \rho(0) | n \rangle \tag{2.32}$$

as has been shown by Alicki[4] for a finite temperature bath.

III. DAMPED HARMONIC OSCILLATOR WITHOUT THE ROTATING-WAVE APPROXIMATION

We consider a harmonic oscillator coupled to a reservoir with a coordinate-coordinate coupling. This is described by the Hamiltonian

$$H = \hbar\omega a^\dagger a + \sum_j \hbar\omega_j a_j^\dagger a_j + \hbar \sum_j [g_j a_j^\dagger (a + a^\dagger) + \text{h.c.}] \ , \tag{3.1}$$

where g_j is the coupling constant. A comprehensive analysis of the problem has been given by Agarwal[11] and we shall make use of his result. The equation for the Wigner function of the oscillator interacting with a finite temperature bath is[11]

$$\frac{\partial W(p,q)}{\partial t} = -\frac{\partial}{\partial q}\left[\frac{p}{m} W\right] + \frac{\partial}{\partial p}[(m\omega^2 q + 2\kappa p) W] + 2m\hbar\omega\kappa(\bar{n} + \tfrac{1}{2}) \frac{\partial^2 W(p,q)}{\partial p^2} \ , \tag{3.2}$$

where

$$\bar{n} = \frac{1}{\exp[(\hbar\omega/kT) - 1]}$$

and $\kappa = \pi f(\omega) |g(\omega)|^2$ where $f(\omega)$ is the density of bath oscillators, and p and q are the canonical variables for momentum and position. The above equation for the Wigner function follows from the master equation derived in the Born and Markov approximations. We note that it has the same form as Eq. (5.14) of Caldeira and Leggett,[12] where it appears as the high-temperature, weak-coupling limit of their general formulation.

Consider the harmonic oscillator initially in a coherent state $|z_0\rangle$, where

$$z_0 = \frac{1}{\sqrt{2\hbar}}\left[(m\omega)^{1/2} x_0 + \frac{i p_0}{(m\omega)^{1/2}}\right] . \tag{3.3}$$

The solution for the Wigner function at time t is[11]

$$W(q,p,t) = (2\pi^2\Delta)^{-1/2} \times \exp\left[-\frac{1}{2\Delta}\{\beta[x - \langle x(t)\rangle]^2 + \alpha[p - \langle p(t)\rangle]^2 - 2\gamma[x - \langle x(t)\rangle][p - \langle p(t)\rangle]\}\right] , \tag{3.4}$$

where

$$\langle x(t)\rangle = \left[\left[\cos(\omega_0 t) + \frac{\kappa}{\omega_0}\sin(\omega_0 t)\right] x_0 + \frac{\sin(\omega_0 t)}{m\omega_0} p_0\right] e^{-\kappa t} ,$$

$$\langle p(t)\rangle = \left[\left[\cos(\omega_0 t) - \frac{\kappa}{\omega_0}\sin(\omega_0 t)\right] p_0 - m\frac{\omega^2 \sin(\omega_0 t)}{\omega_0} x_o\right] e^{-\kappa t} ,$$

$$\alpha = \frac{\hbar\bar{n}}{m\omega}\left[1 - \left[\frac{\omega^2}{\omega_0^2} - \frac{\kappa^2}{\omega_0^2}\cos(2\omega_0 t) - \frac{\kappa}{\omega_0}\sin(2\omega_0 t)\right] e^{-2\kappa t}\right] + \frac{\hbar m\omega}{2} , \tag{3.5}$$

$$\beta = \hbar\bar{n} m\omega\left[1 - \left[\frac{\omega^2}{\omega_0^2} - \frac{\kappa^2}{\omega_0^2}\cos(2\omega_0 t) - \frac{\kappa}{\omega_0}\sin(2\omega_0 t)\right] e^{-2\kappa t}\right] + \frac{\hbar m\omega}{2} ,$$

$$\gamma = \hbar 2\kappa\omega\bar{n}\frac{\sin^2(\omega_o t)}{\omega_0^2} e^{-2\kappa t} ,$$

$$\omega_0 = (\omega^2 - \kappa^2)^{1/2}, \quad \Delta = (\alpha\beta - \gamma^2) \ .$$

The Wigner function may be expressed as the Fourier transform of the off-diagonal matrix elements of the density operator in the coordinate representation

$$W(x,p,t) = \frac{1}{2\pi\hbar}\int \exp\left[\frac{ipy}{\hbar}\right] \langle x - \tfrac{1}{2}y | \rho | x + \tfrac{1}{2}y\rangle dy \ . \tag{3.6}$$

Thus $\langle x - \frac{1}{2}y | \rho | x + \frac{1}{2}y\rangle$ may be obtained from the inverse Fourier transform of the Wigner function. Let us first consider the long-time limit where

$$\alpha = \frac{\hbar(\bar{n} + \frac{1}{2})}{m\omega} , \quad \beta = \hbar(\bar{n} + \tfrac{1}{2}) m\omega ,$$

$$\gamma = 0 , \quad \Delta = \alpha\beta ,$$

$$\langle x(t)\rangle = \langle p(t)\rangle = 0 \ .$$

Thus

$$W(x,p,\infty)=(2\pi^2\alpha\beta)^{-1/2}\exp\left[-\frac{1}{2}\left[\frac{x^2}{\alpha}+\frac{p^2}{\beta}\right]\right]. \tag{3.7}$$

The off-diagonal matrix element in the coordinate representation is

$$\langle x-\tfrac{1}{2}y\,|\,\rho\,|\,x+\tfrac{1}{2}y\rangle=N\exp\left[\frac{-x^2}{2\sigma_x^2}\right]\exp\left[\frac{-y^2}{2\sigma_y^2}\right]. \tag{3.8}$$

The diagonal matrix elements of ρ, that is, the weight function for the center-of-mass coordinate x is a Gaussian with width

$$\sigma_x^2=\frac{\hbar(\bar{n}+\frac{1}{2})}{m\omega}. \tag{3.9}$$

The off-diagonal matrix elements also have a Gaussian distribution with width

$$\sigma_y^2=\frac{\hbar}{4(\bar{n}+\frac{1}{2})m\omega}. \tag{3.10}$$

In a similar fashion one may derive the matrix elements in the momentum representation $\langle p-\frac{1}{2}r\,|\,\rho\,|\,p+\frac{1}{2}r\rangle$. In this case, we find for the widths of the diagonal (σ_p^2) and off-diagonal matrix elements (σ_r^2)

$$\sigma_p^2=\hbar m\omega(\bar{n}+\tfrac{1}{2}), \qquad \sigma_r^2=\frac{\hbar m\omega}{4(\bar{n}+\frac{1}{2})}. \tag{3.11}$$

We shall now consider two limiting cases of these expressions. In the low-temperature limit ($T=0$, $\bar{n}=0$)

$$\sigma_x=\sigma_y=\left[\frac{\hbar}{2m\omega}\right]^{1/2}, \qquad \sigma_p=\sigma_r=\left[\frac{\hbar m\omega}{2}\right]^{1/2}. \tag{3.12}$$

Thus at $T=0$ the harmonic oscillator is in a minimum uncertainty state ($\sigma_x\sigma_p=\hbar/2$), the ground state. Note that $\sigma_y\sigma_r=\hbar/2$, although σ_y and σ_r do not represent the uncertainties in any canonically conjugate pair of observables.

In the high-temperature limit $T\to\infty$, $\bar{n}\to kT/\hbar\omega$, and

$$\sigma_x^2=\frac{KT}{m\omega^2}, \quad \sigma_p^2=mkT \tag{3.13}$$

which are the characteristic scales of the Boltzman distribution, and

$$\sigma_y^2=\frac{\hbar^2}{4mkT}, \quad \sigma_r^2=\frac{m(\hbar\omega)^2}{4kT}. \tag{3.14}$$

Here the characteristic length scale σ_y is $\sqrt{2/\pi}$ times the de Broglie wavelength.

Analogous results have been found for the free particle.[13] We note that the product of the variances in the off-diagonal terms is

$$\sigma_y\sigma_r=\frac{\hbar}{2}\frac{\hbar\omega}{2kT}. \tag{3.15}$$

While we emphasize that this is not an uncertainty relation for a canonically conjugate pair of observables, it is suggestive to view it as characterizing the competition between quantum ($\hbar\omega\sim 2kT$) and classical behavior ($2kT\gg\hbar\omega$).

The high-temperature bath destroys the quantum correlations present in the initial coherent state, any residual quantum correlations being on the scale of the de Broglie wavelength. The final state of the system may be described by a Gaussian mixture of position eigenstates with a variance given by σ_x^2.

We shall now investigate how quickly the system reaches the mixed state. To do this, we use the solution for the time-dependent Wigner function given by Eq. (3.4). We shall simplify this expression by neglecting terms in κ/ω_0, in which case the Wigner function assumes the form

$$W(x,p,t)=\frac{1}{2\pi^2\alpha\beta}\exp\left[-\frac{1}{2}\frac{[x-\langle x(t)\rangle]^2}{\alpha}-\frac{1}{2}\frac{[p-\langle p(t)\rangle]^2}{\beta}\right], \tag{3.16}$$

where

$$\alpha=\frac{\hbar\bar{n}}{m\omega}(1-e^{-2\kappa t})+\frac{\hbar}{2m\omega}, \qquad \beta=\hbar\bar{n}m\omega(1-e^{-2\kappa t})+\frac{\hbar m\omega}{2}. \tag{3.17}$$

The time-dependent behavior of the off-diagonal matrix element is

$$\langle x-\tfrac{1}{2}y\,|\,\rho(t)\,|\,x+\tfrac{1}{2}y\rangle=N\exp\left[-\frac{[x-\langle x(t)\rangle]^2}{2\sigma_x^2(t)}\right]\exp\left[-\frac{[y-\langle y(t)\rangle]^2}{2\sigma_y^2(t)}\right], \tag{3.18}$$

where

$$\sigma_x^2(t)=\frac{\hbar\bar{n}}{m\omega}(1-e^{-2\kappa t})+\frac{\hbar}{2m\omega},$$

$$[\sigma_y^2(t)]^{-1}=\frac{4\bar{n}m\omega}{\hbar}(1-e^{-2\kappa t})+\frac{2m\omega}{\hbar}.$$

Initially the deviations from the mean are given by the width of a coherent state. Thus for a high-temperature bath the off-diagonal matrix elements decay as $2kT/\hbar\omega(1-e^{-2\kappa t})$. Clearly, for $kT\gg\hbar\omega$ this implies a rapid decay of the off-diagonal terms. On the other hand, the width of the diagonal matrix elements spreads from the initial coherent state at a rate given by $2kT/\hbar\omega(1-e^{-2\kappa t})$.

IV. SUMMARY

In this paper we have analyzed the effect of dissipation on a macroscopic superposition of quantum states using a Markovian master-equation treatment of the dissipation. We showed that the quantum coherence of a superposition of two position eigenstates of a harmonic oscillator decays at a rate proportional to the product of the square of the difference between the two initial positions and the energy relaxation rate. This underlines the difficulty of observing a macroscopic quantum state. It also has implications in the quantum theory of measurement where the role of the environment is played by the measuring apparatus. A pure state is rapidly reduced to a mixture in the pointer basis of the system via the interaction with the measuring apparatus. We considered a harmonic oscillator initially in a coherent state interacting with the environment via a coordinate-coordinate coupling. For a high-temperature bath the only remaining quantum correlations in the steady state are on the scale of the de Broglie wavelength.

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